having two strings to our bow; it is at any rate worth urgently raising the question, and speaking before it is too late. On one point my graphical investigation has led me to a consideration which at first sight, at any rate, seems to be in favour of Delisle's method as against Halley's, and it is this: if there is any error in the declination of *Venus*, or the measure of the semidiameters, it would entail a greater error on Halley's than on Delisle's method. As far as I have thus far made out, an error of 1^s in these measures would affect the time-path to the extent of about 9^s for Halley's method, and about 5^s for Delisle's; but this is perhaps only an additional reason why the two methods should be resorted to, if only for the sake of clearing up any such possible error, or errors.

April 9, 1873.

Graphical Method for Determining the Motion of a Body in an Elliptic Orbit under Gravity. By Richard A. Proctor, B.A., Cambridge.

The student of astronomy often has occasion to determine approximately the motion of bodies, as double stars, comets, meteor systems, and so on,—in orbits of considerable eccentricity; and therefore a graphical method for solving such problems in a simple yet accurate manner will probably be of use to many readers of the *Notices.* The process now to be described is, so far as I am aware, a new one, (though I have an indistinct recollection of a paper, by Mr. Waterston, if I remember rightly, suggesting that part of the construction which relates to the curve of sines).* Of course it involves no new principles. By its means a figure, such as the lithographed diagram illustrating this paper, having once for all been carefully inked in on good drawing card, the motion of a body in an orbit of any eccentricity can be determined by a pencilled construction of great simplicity, which can be completed (including the construction of the ellipse) in a second or two.

Let A P A′ be an elliptical orbit of which A C A′ is the major axis, S being the centre of force, so that A is the aphelion, and A′ the perihelion. Let a be the half major axis; e the eccentricity C S; H half the periodic time, and T the time in which the body moves from A to P.

On A A′ describe the auxiliary semicircle A b A′.

Then

$$\begin{aligned} T &\propto \text{area } ASP \propto \text{area } ASQ \\ &\propto \text{area } (ACQ + SCQ) \\ &\propto a\,(AQ) + e\,(QM) \\ &\propto AQ + \frac{e}{a}\,.\,QM. \qquad (\alpha) \end{aligned}$$

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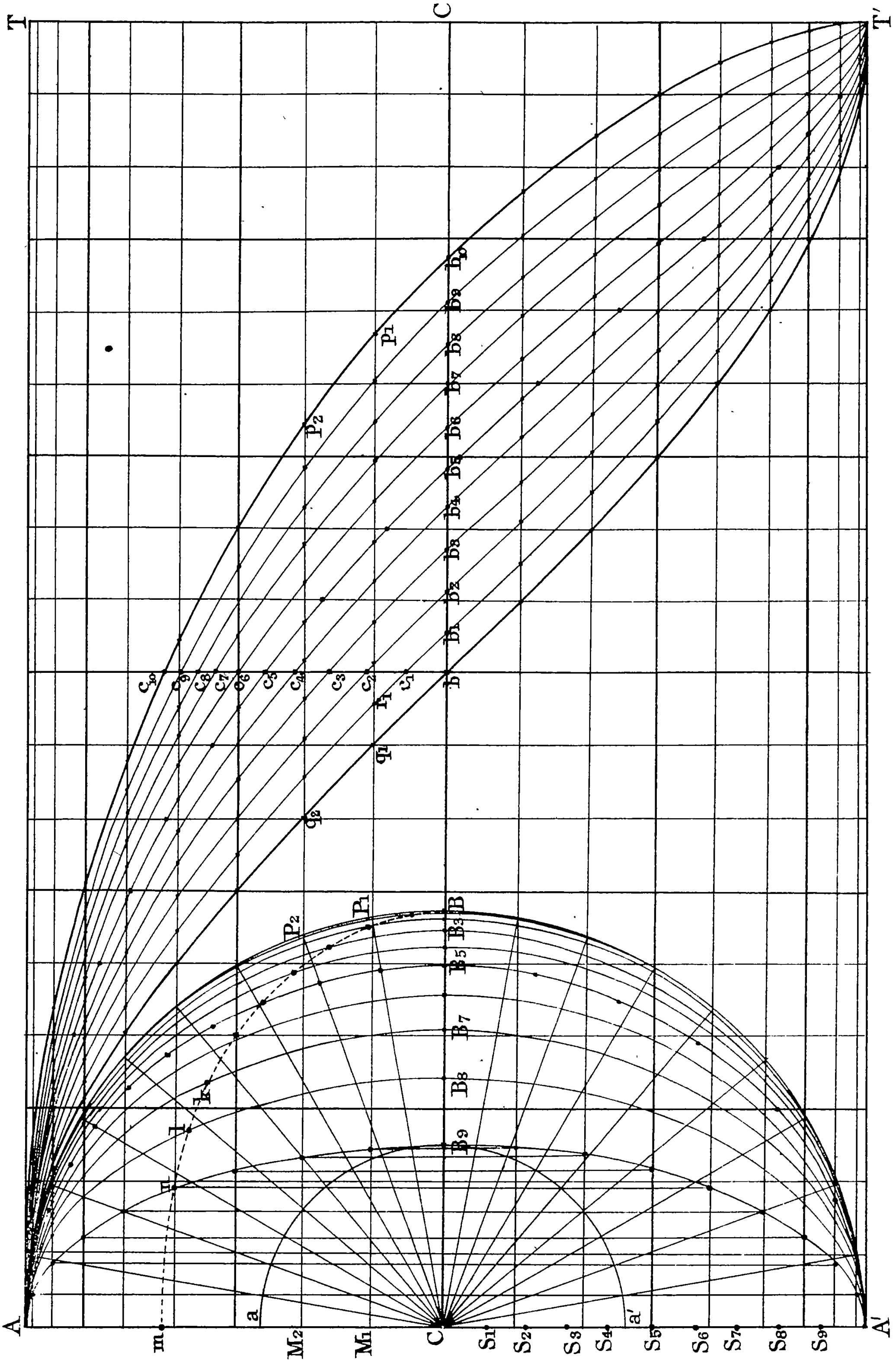
* See addendum to present paper.


T
C′
T′
A
A′
C
a
a′
m
M2
M1
S1
S2
S3
S4
S5
S6
S7
S8
S9
B
B3
B5
B7
B8
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P1
P2
b
b1
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b3
b4
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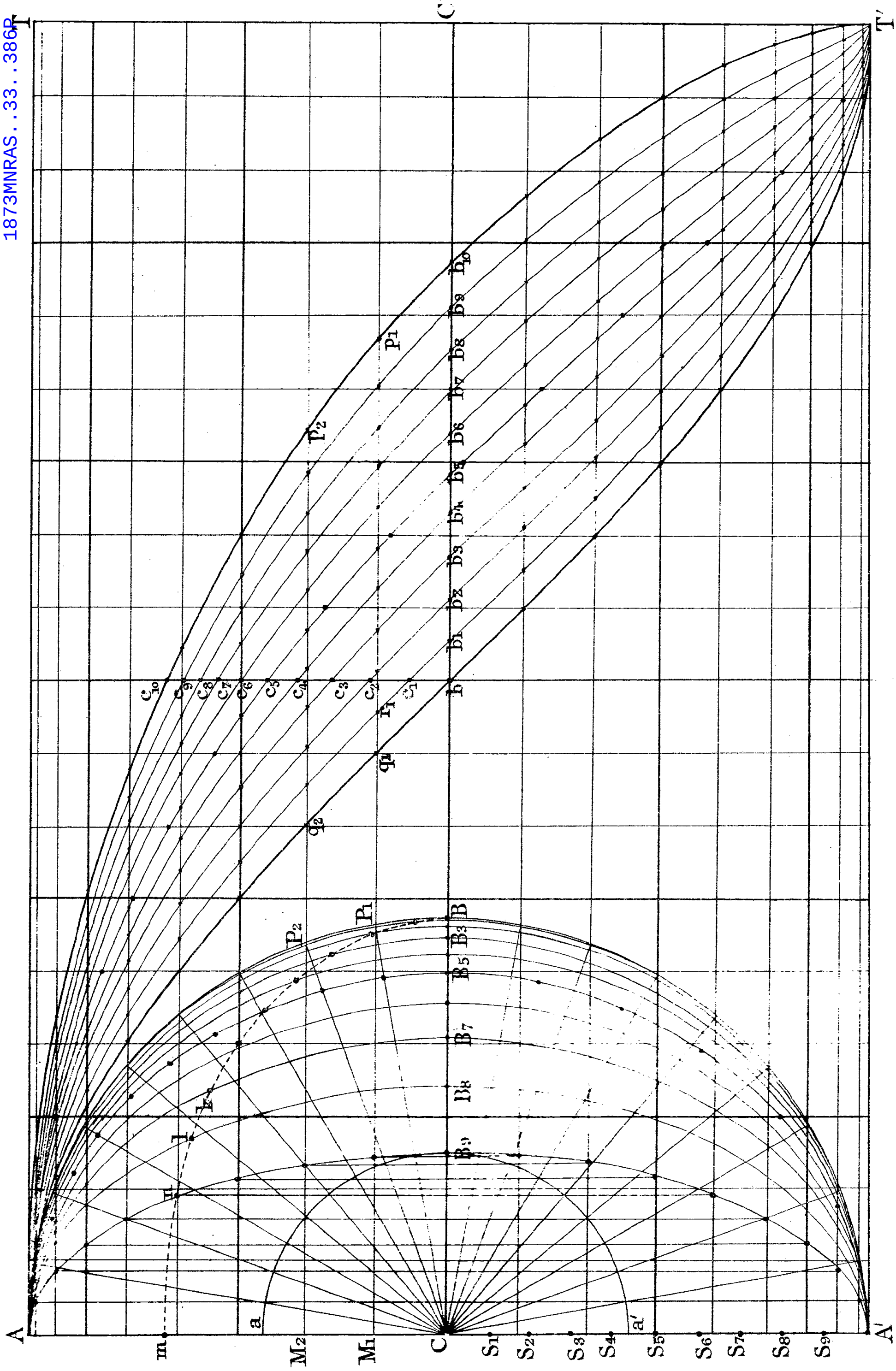
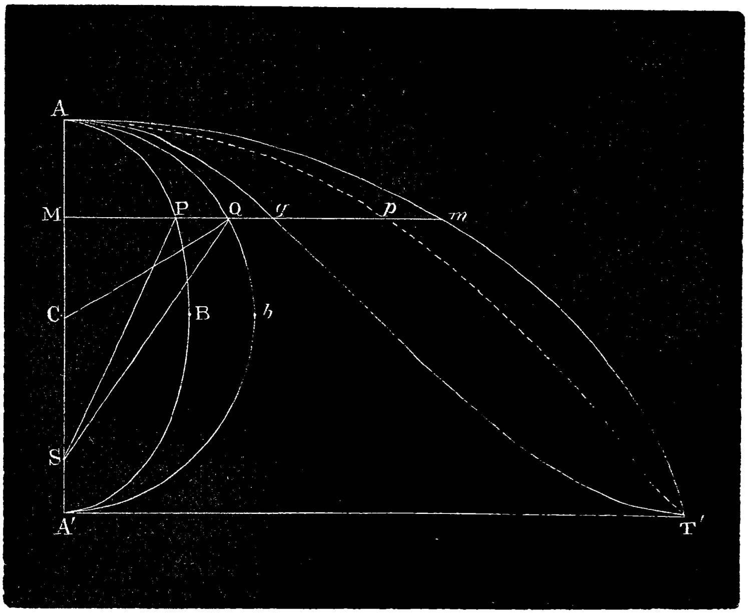


A
T
C′
T′
A′
C
B
B3
B5
B7
B8
B9
P1
P2
a
a′
m
M2
M1
S1
S2
S3
S4
S5
S6
S7
S8
S9
b
b1
b2
b3
b4
b5
b6
b7
b8
b9
b10
c1
c2
c3
c4
c5
c6
c7
c8
c9
c10
p1
p2
q1
q2
r1

Now if A m T′ be a cycloid having A A′ as its diameter, then

$$\text{Ordinate M}\,m = \text{A Q} + \text{Q M}$$

And if we take M q = A Q, we have q a point on A q T′, the curve of sines, otherwise called the companion to the cycloid.



The line $q\,m$ is then equal to Q M; and if we take a point p on m Q such that

$$q\,p : q\,m :: \text{C S} : \text{C A}$$

we have

$$\text{M}\,p = \text{A Q} + \frac{e}{a}\,\text{Q M}.$$

accordingly we see (from α) that we may represent the time in traversing the arc A P by the ordinate M p to a curve A p T′, obtained by dividing all such lines as $q\,m$ (joining the cycloid and its companion, and paralled to A′ T′) so that $q\,p : q\,m$ as e to a.

Accordingly, if we construct such a diagram as is shown in the illustrative plate, in which A T′ is a semi-cycloidal arc and A b T its companion, while intermediate curves are drawn dividing all such lines as $b\,b_{10}$ into ten or any other convenient number of equal parts, the curves through the successive points b, b_1, b_2, &c., to b_{10}, give us the time-ordinates for bodies moving in ellipses having A and A′ as apses, and their centres of force respectively at C, S_1, S_2, S_3, &c. . . . S_9, and A′.

In the plate the semi-ellipses corresponding to these positions of the centre of force are drawn in, and it will be manifest that any ellipse intermediate to those shown can be pencilled at once, with sufficient accuracy. Ellipses within A B_9 A′ have their focus of force between S_9 and A′, and are exceptionally

eccentric.* It is easy to construct such an ellipse, however, in the manner indicated for the semi-ellipse A B_9 A′. For the radial lines and the parallels to A T through their extremities are supposed to be inked in; and (taking the case of ellipse A B_9 A′) we have only to draw the semicircle a B_9 a', and parallels to A A′ through the points where the radial lines intersect this semicircle, to obtain by the intersections of these parallels with the parallels to A T a sufficient number of points on the semi-ellipse.

The illustrative diagram has been specially constructed for the use of those who may have occasion to employ the method, and will be found sufficiently accurate for all ordinary purposes. Before proceeding, however, to show how the method is applied in special cases, I shall describe how such a diagram should be constructed :—

First the semicircle A B A′ must be drawn, and the lines A T, A′ T′ square to A A′. Then C A′ must be divided into ten equal parts (and when the figure is large, a plotting-scale for hundredths, &c. should be drawn). Next A′ T and A T must be each taken equal to 3·1416 where C A′ is the unit. Join T T′. Now A T and A′ T′ represent, as time-ordinates, the half-period of any body moving in an ellipse having A A′ as major axis. Each must now be divided into the same number of equal parts, and it is convenient to have eighteen such parts. (So that in the illustrative case of our Earth, three divisions represent a month.) Next the semicircle A B A′ must be divided into eighteen equal parts. Through the points of division on the semicircle parallels to A T, A′ T′, are to be drawn,† and the points of division along A T and A′ T′ are to be joined by parallels to A A and T T′. Then the curve A b T′, the "companion to the cycloid," runs through the points of intersection of the first parallel to A T and the first to A A′, the second parallel to A T and the second to A A′, the third parallels to these lines, the fourth, and so on. We have now only to take $b\,b_{10}$ equal to C B ; $q_1 p_1$ equal to M_1 P_1 ; $q_2 p_2$ equal to M_2 P_2 ; and so on, to obtain the required points on the cycloid A b_{10} T′ ; and the equidivision of all such lines as $b\,b_{10}$, $q_1 p_1$, $q_2 p_2$ (into ten parts in the illustrative diagram) gives us the required points on the intermediate curves.

Now let us take some instances of the application of the diagram.

I. Suppose we wish to divide a semi-ellipse of given eccentricity into any given number of parts traversed in equal times, and let the eccentricity be $\frac{1}{2}$, and 18 the given number of parts ‡ :—

* It is manifest that when the centre of force is at A′ we have the case of a body projected directly from a centre of force, and the time-curve becomes the cycloid A b_{10} T′. Thus the above lines give a geometrical demonstration of the relation established analytically in my paper in the *Monthly Notices* for November 1871.

† Practically it is convenient to draw another semicircle on T T′, divide its circumference into eighteen parts, and join the corresponding points of division on the two semicircles.

‡ This selection is made solely to avoid the addition of lines and curves not necessary to the completeness of the diagram.

Then S_5 is the centre of force; A B_5 A′ the semi-ellipse; and A b_5 T′ the time-curve. Now the dots along A b_5 T′ give the intersection of the time-curve with the time-ordinates parallel to A A′, and therefore parallels to A T, though these dots (not drawn in the figure, to avoid confusion) indicate by their intersection with the semi-ellipse A B_5 A_1 the points of division required.

II. Suppose we wish to know how far the November meteors travel from perihelion in the course of one quarter of their period, that is, one half the time from perihelion to aphelion:—

The curve A B_9 A_1 is almost exactly of the same eccentricity as the orbit of the November meteors. To avoid additional lines and curves, let us take it as exactly right. Then A b_9 T′ is the time-curve. For the quarter period from perihelion (or aphelion), we take of course the middle vertical line, which intersects A b_9 T′ in c_9. This point by a coincidence is almost exactly in a parallel to A T, and this parallel meets the semi-ellipse A B_9 A′ in n, the required point on the orbit. In other words, the journey of the November meteors from A to n occupies the same time as their journey from n to A′, S_9 being the position of the Sun, and the Earth's distance from the Sun approximately equal to A′ S_9.

III. Suppose we require, in like manner, the quarter-period positions in different orbits, all having A A′ as major axis, but their centres of force variously placed along C A′. We get any number of points, n, l, k, precisely as n was obtained; m, of course, is on the parallel through C_{10}; and we obtain, in fine, the curve $m\, n\, l\, k$ B, which resembles, but is not, an elliptic quadrant.

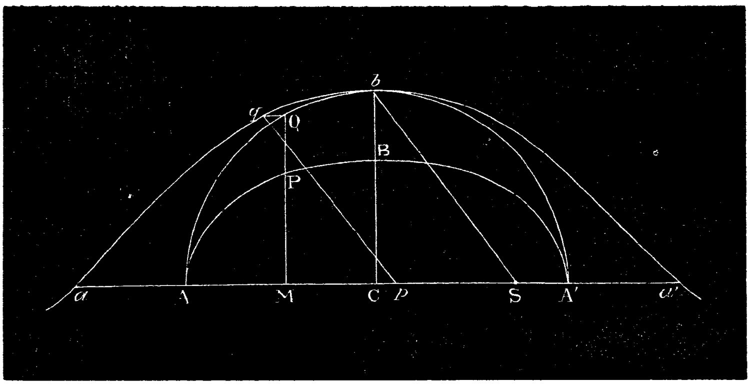
IV. Suppose we require to know in what time the half orbit from aphelion or perihelion is described in orbits of different eccentricity. The required information is manifestly indicated by the intersection of C C′ with the time-curves, in b, b_1, b_2, &c. Thus in the circle, A B is described in the time represented by C b; in the semi-ellipse A B_3 A′, A B_3 is described in the time represented by C b_3, and B_3 A′ in the time represented by C′ b_3; and so on for the other semi-ellipses.

V. Suppose we require to determine approximately the "equation of the centre" for a body when at any given point of its orbit of known eccentricity. Take the case of *Mars*, whose eccentricity being nearly $\frac{1}{10}$, his path is fairly represented by the ellipse next within A B A′, and his time curve by A b_1 T′. Then the equation of the centre, when *Mars* is at his mean distance, is represented by $b\, b_1$; when *Mars* is at P_1 (not on the circle, but on the curve just within), the equation of his centre is represented by $q_1\, r_1$; and so on.

Many other uses and interpretations of the time-curves will suggest themselves readily to those who are likely to use the diagram.

After the above method had been briefly described, Prof.

Adams, who was in the chair, mentioned a method (devised by himself many years since) by which the same results can be obtained from a single curve,—the "companion to the cycloid" or "curve of sines." I have only Prof. Adams's *vivâ voce* sketch to guide me; but believe that I am right in saying that his method may be thus exhibited:—Let $a\,b\,a'$ be the y-positive half of one wave of the "curve of sines," b C its diameter, A b A′ semicircle with radius b C. Let A B A′ be a half-ellipse having focus at S.



Then time in any arc A P of this ellipse may be thus determined. Join b S, produce ordinate P M to Q, draw Qq parallel to $a\,a'$, and $q\,p$ parallel to b S; then does $a\,p$ represent the time in traversing A p, where $a\,a'$ represents the half period. And *vice versâ*, if we require the position of the moving body after any time from the apse, say aphelion, then take $a\,p$ to represent the time where $a\,a'$ is the half period, A C A′ the major axis, S the centre of force; join S b, draw $p\,q$ parallel to S b, q Q parallel to A A′, and Q P perpendicular to A A′ gives P the point required.

It will be manifest that in principle my method is identical with this, for in my figure the time is represented by M p, where M q is equal to the arc A Q, and $q\,p$ is equal to Q M reduced in proportion of C S to C A. Now $a\,p$ in the second figure is the projection of A q and $q\,p$; and the projection of A q is equal to the arc A Q, while the projection of $q\,p$ is equal to Q M reduced in the proportion of C S to A C.

But although Prof. Adams's construction, besides being earlier in publication, has the advantage of requiring but a single curve, yet mine subserves a distinct, and I think useful purpose. In fact, for the particular purpose I had in view, my construction alone avails. We see from the second figure that to give the relation between the times and positions in the case of the ellipse A p A′, we require a series of parallels to b C, $a\,a'$ and b S; and the parallels to b S only serve for this one case. Therefore we could not construct a reference figure for many cases, without having *many series of parallels* and a very confusing result. In my construction we have instead *many curves*, but a result which is not confusing, because each curve is distinct from the rest.

In fact, I may sum up the different qualities of the two constructions by saying that, whereas Prof. Adams's is the proper construction if a problem of the kind has to be dealt with *ab initio*, mine is, I conceive, the proper construction for a reference-figure. I repeat, however, that the principle of my construction cannot be regarded as new. Moreover, as a geometrical expression of the relation between the time and the position in elliptic motion, Prof. Adams's construction is manifestly superior.

On the Rejection of Discordant Observations.

By J. W. L. Glaisher, B.A., Fellow of Trinity College, Cambridge.

In the *Monthly Notices* for April 8, 1868 (vol. xxviii. pp. 165–168), Mr. Stone suggested a criterion for the rejection of discordant observations, which it was the original object of this communication to examine. Before doing so, however, I wish to allude to a passage contained in a paper of mine on the subject of errors of observation, printed in vol. xxxix. of the *Memoirs* of the Society, in which the mode of treatment to which the theory itself leads is noticed, and to explain this method in greater detail than is there done. It will be seen that it supersedes the necessity for the *rejection* of anomalous observations.

The passage referred to occurs on p. 103, and consists of the quotation from De Morgan, and the remarks that precede it. As the extract from De Morgan is the foundation of what follows, and is very short, I here quote it again:—"Assuming the weights as nearly as they can be found, ascertain the most probable result, from which find the weights of the equations. If these agree with the assumed weights, the process is finished; if not, repeat the process with the new weights, and so on, until a result is obtained for which the assumed and deduced weights of the equations are sufficiently near to equality."

Believing this to be the true mode of completing the treatment of observations (and having been independently led to it), I proceed to develope its principles rather more in detail, and to answer some possible objections.

The problem we are concerned with is, Given a number of direct observations of the same quantity, determine the most probable value of that quantity. This is, of course, not the most general way of stating the question, as it is assumed that only one unknown is to be determined; but its discussion involves all the essential principles, and in a form free from unnecessary complication.

The usual reasoning is to suppose that all the observations